

University of Saskatchewan
Department of Physics and Engineering Physics
EP 228.3

Final Examination
A CLOSED BOOK EXAMINATION
[Examiner D.A. Degenstein]

Time: 3 hours.

Date: April, 2005

Instructions: Candidates are to answer *ALL* questions in the booklets provided.

There are 13 questions in the exam.

All questions **DO NOT** have the same value.

Electronic calculators are required.

One formula sheet is allowed.

Please read each question carefully before attempting it.

THINK before you act!!!!

- 1) Express each of the complex expressions in terms of $z = x + iy$. Show some work or you get no marks. (12 marks)

i) $z = \frac{dz_1^2}{dt}$ where $z_1 = 5 - i2t$

ii) $z = \frac{(z_1^* - z_2)}{z_2}$ where $z_1 = 2e^{i\frac{4\pi}{3}}$ and $z_2 = 2e^{-i\frac{\pi}{3}}$

iii) $z(3,0)$ where $z(x,t) = \frac{d^2 e^{z_1}}{dx^2}$ and $z_1 = -i\frac{\pi}{3}t - 2x$

iv) $z = z_1 z_2$ where $z_1 = (8 + i12)^{-1}$ and $z_2 = 2 - i3$

- 2) Show, using the complex representation of $\cos\theta$ and $\sin\theta$, that: (5 marks)

$$[\tan(\theta_1) - \tan(\theta_2)]\cos(\theta_1)\cos(\theta_2) = \sin(\theta_1 - \theta_2)$$

- 3) What are the non-zero frequency components, a_n and b_n values, and the angular frequencies of these components for the function

$$f(t) = 6[\tan(60\pi t) - \tan(20\pi t)]\cos(60\pi t)\cos(20\pi t) - 5\cos(40\pi t) + 1.2\sin(40\pi t) - 3.8$$

on the interval $[-0.1, 0.1]$? (5 marks)

- 4) Assume the number of Uranium molecules in an experiment, N_t , can be defined at any time as $N_t = N_{t-1}r - A$. In this expression A and r are constants and N_{t-1} is the number of Uranium molecules at the previous time step. Prove by induction that the number of Uranium molecules in the system at any time t is given by

$$N_t = N_0 r^t - A \frac{(1 - r^t)}{(1 - r)}$$

where N_0 is the number of molecules at time $t = 0$ s. (5 marks)

- 5) What are the Fourier coefficients, a_n and b_n , for the function shown in Figure 5.1? Assume $T_0 = -3.0$ s and $T_f = 3.0$ s. (8 marks)

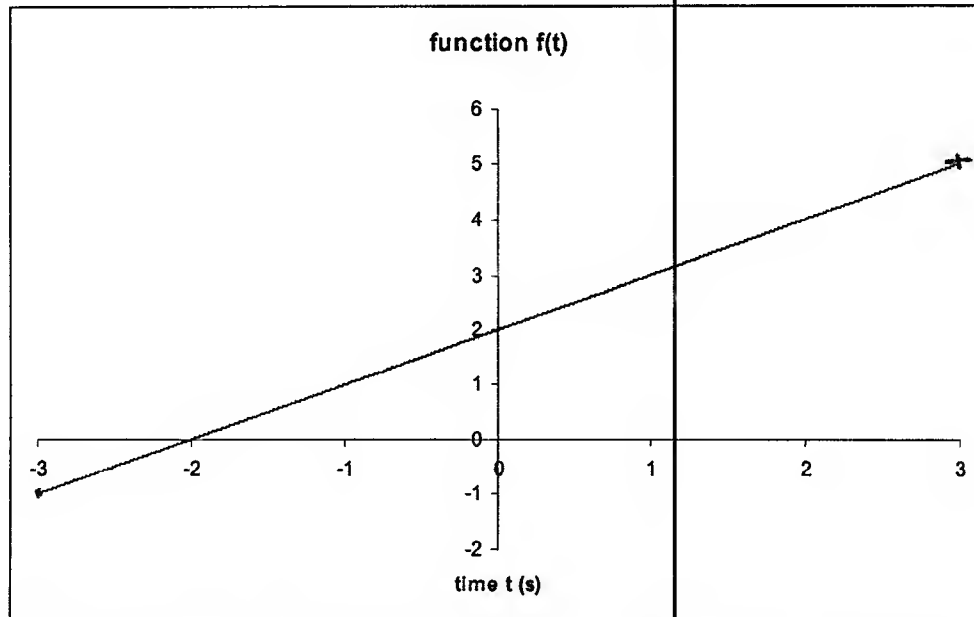


Figure 5.1: Estimate the required points by reading them from the graph.

- 6) Given the matrix $A = \omega \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ and the definition
$$e^{At} = I + \frac{At}{1!} + \frac{A^2 t^2}{2!} + \frac{A^3 t^3}{3!} + \frac{A^4 t^4}{4!} + \dots$$
 show that
$$e^{At} = \begin{pmatrix} \cos(\omega t) & -\sin(\omega t) \\ \sin(\omega t) & \cos(\omega t) \end{pmatrix}.$$
 (6 marks)

- 7) At a height h above the surface of the Earth the gravitational force on a mass m is given by

$$F = \frac{mMG}{(R+h)^2} \quad (7.1)$$

where M and R are respectively the mass and radius of the Earth and G is Newton's gravitational constant. For small values of h it is common to use the approximation

$F = mg$ where $g = \frac{MG}{R^2}$. If more precision is required for a particular calculation

what are the next two correction terms in the Taylor series expansion of equation 7.1. (5 marks)

- 8) Given the matrix $A = \begin{pmatrix} -3 & -1 \\ 1 & 0 \end{pmatrix}$: (10 marks)

- What is the determinant of A ?
- What is the inverse of A ?
- What is x in $Ax = \begin{pmatrix} 2 \\ i \end{pmatrix}$?
- What are the eigenvalues of A ?
- What are the eigenvectors of A ?

- 9) Assume $p = (-5, 2)$ is a point in a two-dimensional Cartesian coordinate system represented by the two unit vectors (\hat{x}, \hat{y}) then: (9 marks)

- i) Given the vectors $\bar{v}_1 = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ and $\bar{v}_2 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$ what are the constants that multiply \bar{v}_1 and \bar{v}_2 such that the point p is a linear combination of \bar{v}_1 and \bar{v}_2 ?

- ii) Given the vectors $\bar{v}_1 = \begin{pmatrix} -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{pmatrix}$ and $\bar{v}_2 = \begin{pmatrix} -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{pmatrix}$ what are the constants that multiply \bar{v}_1 and \bar{v}_2 such that the point p is a linear combination of \bar{v}_1 and \bar{v}_2 ?

- iii) The vectors \bar{v}_1 and \bar{v}_2 in part ii) represent a coordinate system that is rotated by a particular angle with respect to the original (\hat{x}, \hat{y}) coordinate system. What is the value of the vector p , in the (\hat{x}, \hat{y}) coordinate system, if it is rotated by the same amount as \bar{v}_1 and \bar{v}_2 ?

$$x^3 + 3x^2(-3) + 3x(9) + (-3)$$

- 10) Take a Taylor Series expansion of $f(x) = -2 + 3(x-3) - 8(x-3)^2 + 2.5(x-3)^3$ about the point $x=3$. What are the first three non-zero coefficients of this expansion? What is the value of the MacLaurin series expansion of this function evaluated at $x=4$? (5 marks)

- 11) Given the following data set : (9 marks)

X	f(x)
-10	-26
-5	14
0	4
5	-56
10	-166

Table 11.1

- Find the coefficients of the quadratic $f(x) = a_2x^2 + a_1x + a_0$ that goes through the two end-points and the middle point of this data set.
 - What are the values of this quadratic at $x = -5$ and $x = 5$?
 - What are the best approximations for the numerical derivatives at each of the relevant data points?
- 12) If z_1 and z_2 are two complex numbers, as seen in Figure 12.1 below, show that $z_1z_2^* + z_1^*z_2 = 2|z_1||z_2|\cos\theta$. Also, use this result and Figure 12.2 to prove, $|z_3|^2 = |z_1|^2 + |z_2|^2 - 2|z_1||z_2|\cos\theta$, the Law of Cosines. (6 marks)

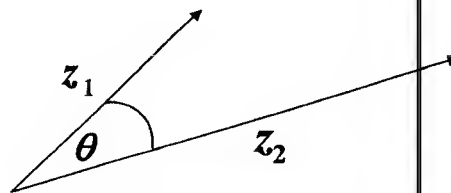


Figure 12.1

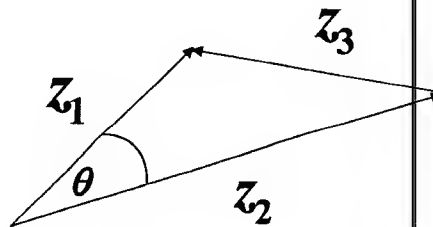


Figure 12.2

13) Given the following data :

x	$f(x)$	$f'(x)$
2	9	
3		6
4		1
5		-7
6		-1
7	13	

where the derivative was taken using the central difference method what is the original data set? (5 marks)

- End of Examination -